Chapter 26
Energy and Momentum Correction Coefficients in Compound Open Channel Flow

Sarjati Sahoo, Jnana Ranjan Khuntia, Kamalini Devi, and Kishanjit Kumar Khatua

Abstract Open channels have a wide range of applications directly or indirectly in our day-to-day life. Irrigation canals, waste water channels, sewers etc. are some of the practical examples of free-surface flow channels. Study of the hydraulic properties of open channel is a prime part of river engineering. This article presented a novel approach to model the two energy correction coefficients (kinetic energy and momentum coefficients) by collecting experimental data sets on rough asymmetric channels. In the present work, two sets of experiments were conducted on a compound asymmetric channel having two types of roughness material on the flood plain (plastic mat and gravel). To develop the models a wide range of data sets has been considered which includes asymmetrical compound channels with smooth or differential roughness cases. For the current work, multivariate regression analysis was used to develop ameliorated models to anticipate error-free values of the energy correction factors. Multivariate regression analysis is used to predict dependent variables from a number of independent variables. In this present study, for implementing multivariate regression analysis, the two correction factors (Energy correction factor α and momentum correction factor β) were chosen as dependent variable and aspect ratio ($\delta = b/h$), roughness ratio ($\gamma = n_{fp}/n_{mc}$), Reynolds No. (R_e) and Froude's No. (F_r), width ratio ($\alpha_1 = B/b$), relative depth ($Dr = (H - h)/H$) are taken as independent variable. Error analysis was performed to determine the accuracy of the present models.

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26.1 Introduction

Open channels are characterized by existence of a free surface. Open channels include both natural rivers and streams as well as man-made structures like irrigation canals, sewers and laboratory flumes. During flood, as the main-channel cannot accommodate all the water inside it, the excess water inundates the flood plain. When this condition arises, it is called as a compound channel. Open channels are used for irrigation, hydropower, drainage systems etc. Therefore, it is necessary to continuously investigate the open channel flow to achieve a more effective design (Khuntia et al. 2018; Devi et al. 2021). So, to design an open channel, three basic laws of conservation such as conservation of mass, energy and momentum are applied. But for free surface flow applications, the three conservation laws are expressed in terms of continuity equation, energy equation, and momentum equation respectively. Mean velocity is used in these equations, by assuming the flow is to be steady and uniform as well as constant vertically across the cross-section of flow. However, for open channels, the velocity distribution is not uniform (Al-Khatib and Dmadi 1999; Seckin et al. 2009). Due to this assumption, the final values of the energy, momentum and other related calculations resulted with some error (Wali 2013; Fenton 2005). To nullify this error, two correction factors are introduced which are called as energy and momentum correction factor or co-efficient. When the flow is steady or nearly uniform, the error resulted by the above assumption may be neglected. That means there is no need to introduce any corrections to the values of energy and momentum correction factors ($\alpha = 1$ and $\beta = 1$). But in practical cases, steady and uniform is nearly impossible. That means the values of correction factors are greater than unity and cannot be neglected. So, to have an error free design of open channels, proper assessment of these two factors is very much essential. It's very common in rivers that the flood plain roughness is more than that of main channel roughness, which affects the flow velocities of main channel and flood plain. Due to which the velocity of flow in main channel is more than the velocity of water in flood plain. This difference in flow velocities cause a lot of momentum transfer within the main channel and flood plain. The whole flow becomes very complex as the due to this non uniformity in velocity distribution in lateral direction. In many literatures it has been clearly documented about exchange of momentum by Sellin (1964), Knight and Demetriou (1983). Shape of the velocity distribution curve affects the energy and momentum correction factors. Many researchers like Al-Khatib and Göğüş (1999), Seckin et al. (2009), Mohanty et al. (2012), developed equations to calculate α and β under certain assumptions and conditions. In the present study, improved models have been developed for correction coefficients specifically for asymmetrical compound channels. For more accuracy, both geometric and hydraulic parameters are considered to develop the model.

26.2 Theoretical Analysis

To design an open channel, three basic laws of conservation such as conservation of mass, energy and momentum are applied. But for free surface flow applications, the three conservation laws are expressed in terms of continuity equation, energy equation, and momentum equation respectively. The equations are given below in Eqs. 26.1–26.3 respectively.

Continuity equation

$$Q = A_1U_1 = A_2U_2 = \cdots = A_nU_n \tag{26.1}$$

Energy equation

$$\frac{P_1}{\rho g} + \frac{U_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{U_2^2}{2g} + z_2 + \sum h \tag{26.2}$$

Momentum equation

$$F_r = \rho Q[U_{2x} - U_{1x}] \tag{26.3}$$

where Q is conveyance, A_1, A_2, and U_1, U_2 are area of cross-section and average velocity at Sects. 1 and 2 respectively. Average velocity is used in all the above three equations by assuming the flow as steady and uniform and also constant vertically across the cross-section of flow. But in case of open channel flows, the velocity distribution is non-uniform due to the boundary resistance. These assumptions may result in erroneous data of energy and momentum, which should be corrected while designing hydraulic structures over open channels. To nullify this error, two correction factors are introduced which are called as energy and momentum correction factor or co-efficient.

To estimate these correction coefficients, Eqs. 26.4 and 26.5 are used and the equations are given below.

$$\alpha = \int_0^A \frac{U^3 dA}{U_m^3 A} \tag{26.4}$$

$$\beta = \int_0^A \frac{U^2 dA}{U_m^2 A} \tag{26.5}$$

where
α: energy correction factor,
β: coefficient of momentum correction,

A: total area of cross-section,
U: flow velocity passing through the small area dA and
U_m: mean or average velocity of flow.

26.3 Experimental Setup and Procedures

To perform experiments, a channel was constructed by using mild steel plates and cement concrete in side the Hydraulics Lab of NIT Rourkela. For continuous supply and recirculation of water into the experimental flume, a large reinforced cement concrete (R.C.C) overhead tank and a volumetric tank were constructed at the upstream and downstream side of the flume respectively by using two 10 HP pumps. To reduce the energy of water, a stilling chamber fitted with flow straighteners and also honeycomb structures was provided at the inlet of the flume. In this process, flow uniformity was achieved throughout the length of the flume. Trapezoidal shape was chosen for the main channel cross section with bottom width (b) 0.33 m, height (h) 0.11 m and side slope 1:1 (H:V) over a length of 12 m. Total width of the compound channel is 1.19 m for acquiring width ratio (α_1) 3.6 and the bed slope of the channel was maintained at 0.001 by adjusting wheel and gear arrangement provided at inlet of the channel. Experiments were performed on the asymmetric channel with differential roughness having smooth main channel and rough flood plain, by using plastic mat of thickness 15 mm having manning's n value 0.024 on the flood plain bed. Another set of experiments were performed on the same channel by changing the flood plain roughness from plastic mat to small gravel of Manning's n value 0.02. Main channel roughness was maintained uniform with side slope and different roughness for flood plain to investigate the momentum transfer and secondary current effect in compound channel. Figure 26.1 shows the layout of NITR channels and Photographs of NITR channels used in the present work are shown in Fig. 26.2.

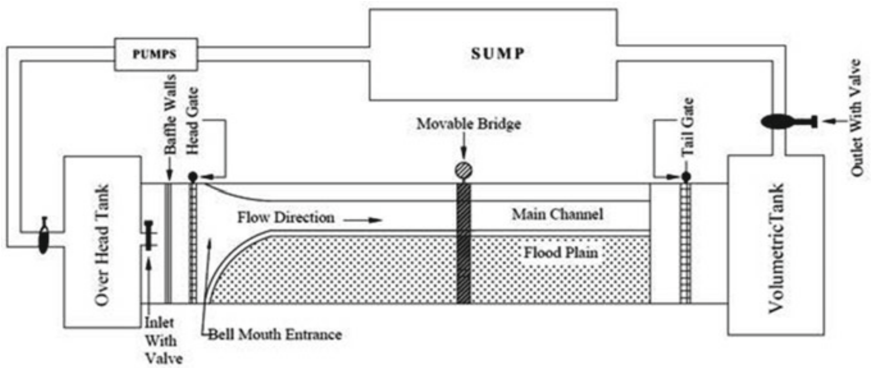

Fig. 26.1 Layout of experimental setup of NITR channels

Fig. 26.2 Photographs of NITR experimental channels used in the present study

26.4 Model Development

26.4.1 Multiple Regression Analysis

In statistics, regression analysis is done to evaluate necessary relation between variables, which includes various techniques for modelling and analysing several variables. Regression analysis helps us to estimate a proper correlation between a response (criterion) variable and one or more regressor variables (or predictors). More precisely, it enables us to perceive the effect of one varying independent parameter on the dependent parameter while the other independent variables are kept constant. Regression analysis is widely used for predicting and forecasting.

Multiple regression analysis is a capable strategy used to anticipate the obscure estimation of a variable from two or more known variables. More specifically, multiple regression analysis helps us to anticipate the estimation of "y" for given estimation of x_1, x_2… x_n. We can likewise concentrate the individual impact of the known factors on the obscure parameter. The standard basis of multiple regression is to know more about the association between a few autonomous or predictor factors and a dependent or paradigm variable. Multiple regression analysis is used to predict dependent variables from a number of independent variables.

In this present study, for implementing multiple regression analysis energy correction factor (α) and momentum correction factor (β) were taken as dependent variable and aspect ratio ($\delta = b/h$), roughness ratio ($\gamma = n_{fp}/n_{mc}$), Reynolds number (R_e), Froude's number (F_r), width ratio ($\alpha_1 = B/b$), relative depth ($D_r = (H - h)/H$) are taken as independent variable. A number of single regression models were developed as one to one relationship between the dependent variable (*DISADF*) and the independent variables (width ratio, aspect ratio, relative depth, friction factor ratio and bed slope). Based on the highest coefficient of regression (R^2) values, best single regression models were selected. The properties of the various channels, which were taken for this study given in Table 26.1.

Table 26.1 Resume of the asymmetrical compound channel data sets

Series	b(m)	B(m)	h(m)	S_c	H(m)	S_0	n_{fp}	n_{mc}	Q(m^3/s)
NIT R1	0.33	1.19	0.11	1	0.134–0.165	0.001	0.024	0.01	0.034−0.0596
NIT R2	0.33	1.19	0.11	1	0.14–0.181	0.001	0.02	0.01	0.0402−0.0820
FCF S06	1.5	4.05	0.15	1	0.15826–0.30185	0.001027	0.01	0.01	0.2235−0.9292
Al-Khatib et al. (2013)	0.1	0.3	0.02	0	0.049–0.110	0.0025	0.015	0.015	0.0033−0.0144
Bousmar (2002)	0.4	0.8	0.05	0	0.0543–0.0912	0.00099	0.0107	0.0107	0.00782−0.02204
Atabay (2001)	0.398	0.8053	0.05	0	0.0593–0.1095	0.002024	0.0091	0.0091	0.015−0.0554
Devi and Khatua 1 (2017)	0.33	1.19	0.11	1	0.12–0.14	0.001	0.01	0.01	0.02873–0.04055
Devi and Khatua 2 (2017)	0.33	1.45	0.11	1	0.12–0.13	0.001	0.01	0.01	0.031461−0.03939
Devi and Khatua 3 (2017)	0.33	1.683	0.11	1	0.125–0.15	0.001	0.01	0.01	0.03024−0.06169
James and Brown T5 (1977)	0.1778	0.4696	0.0508	1	0.05578–0.08016	0.001–0.003	0.012	0.011	0.00708−0.01467
James and Brown T7 (1977)	0.1778	0.8509	0.0508	1	0.05456–0.08168	0.001–0.003	0.011	0.01	0.00558−0.01433
Sun SRC (2007)	0.12	0.27	0.036	0	0.0466–0.0713	0.001	0.0103	0.0103	0.00215−0.00589
Sun STC (2007)	0.12	0.306	0.036	1	0.0475–0.0723	0.001	0.011	0.011	0.00194−0.00558

26.5 Results and Discussions

By using the Eqs. 26.4 and 26.5, energy and momentum correction factors were calculated for the present experimental asymmetric compound channel having two types of roughness on flood plain (Plastic mat- roughness-1, small gravel- roughness-2) and presented in the Table 26.2.

The graphs between energy and momentum correction factors and relative depth for both roughness cases were also plotted and given in Figs. 26.3 and 26.4.

From Figs. 26.3 and 26.4 and Table 26.2, it has been observed that the magnitudes of both the correction factors (α and β) decrease when the depth of flow increases. The transfer of momentum between the main channel and flood plain becomes very intense in case of low flow depth conditions due more difference in flow velocities of main channel and flood plain. But while considering high flow depth cases, this difference becomes less. That's why the resulted energy and momentum coefficients were higher for low flow depth cases and nearer to one when the flow depth increased for the present experimental channel.

Table 26.2 Energy and momentum correction factors for the experimental channel

Roughness-1 (NITR 1)			Roughness-2 (NITR 2)		
H(m)	α_1	β_1	H(m)	α_1	β_1
0.134	1.235	1.080	0.14	1.189	1.066
0.14	1.180	1.054	0.145	1.173	1.065
0.147	1.169	1.045	0.156	1.141	1.040
0.1545	1.123	1.028	0.16	1.124	1.039
0.16	1.122	1.032	0.17	1.089	1.017
0.165	1.108	1.028	0.176	1.069	1.014
–	–	–	0.181	1.057	1.002

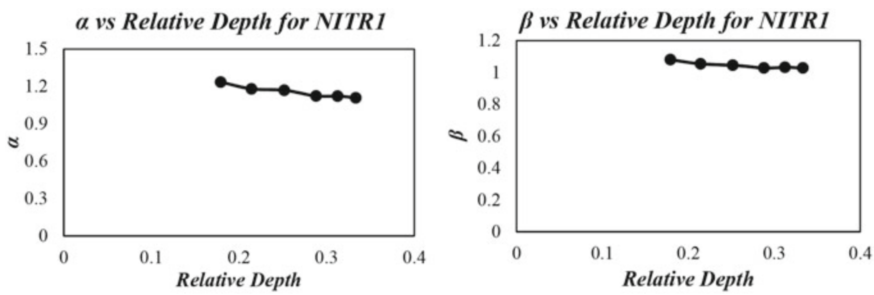

Fig. 26.3 Variation of α and β with relative depth for roughness-1

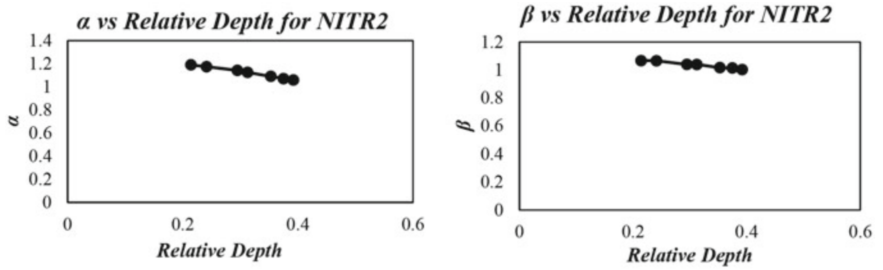

Fig. 26.4 Variation of α and β with relative depth for roughness-2

26.5.1 Regression Models for Asymmetrical Compound Channels

The single regression plots between the dependent parameters (α and β) and independent parameters (δ, γ, R_e, F_r, α_1, D_r) are given in Figs. 26.5 and 26.6.

The relationships between dependent (α) and independent parameters (δ, γ, R_e, F_r, α_1, D_r) for energy correction factor are given in Table 26.3.

The relationships between dependent (β) and independent parameters (δ, γ, R_e, F_r, α_1, D_r) for energy correction factor are given in Table 26.4.

Then multiple regression analysis was applied using single regression relationships between the dependent variables (α and β) and independent parameters (δ, γ, R_e, F_r, α_1, D_r). The proposed models for energy correction factor and momentum correction factor are given by Eqs. 26.6 and 26.7.

$$\alpha = (-0.01\delta) - \left(0.23\gamma^2\right) + (0.54\gamma) - (0.054\ln(R_e)) + \left(0.47F_r^{-0.37}\right) + (0.006\alpha_1) - (0.33D_r) + 1.06 \quad (26.6)$$

$$\beta = (-0.011\delta) - \left(0.155\gamma^2\right) + (0.369\gamma) - (0.00000025R_e) - (0.102F_r) - (0.032\alpha_1) - (0.12D_r) + 1.17 \quad (26.7)$$

26.6 Error Analysis

Error analysis is the study of quantification of error, or uncertainty that may be present in a model. Generally, error analysis is done to determine the strength or accuracy of a model. The different types of errors used to determine the accuracy of a model are Mean Percentage Errors (*MPE*), Mean Absolute Percentage Errors (*MAPE*) and Root Mean Square Errors (*RMSE*). For determining the strength of the model presented in this study, MAPE, MPE and RMSE were used. The errors for various channels taken for this model were calculated using the formula given by Eqs. 26.8–26.10.

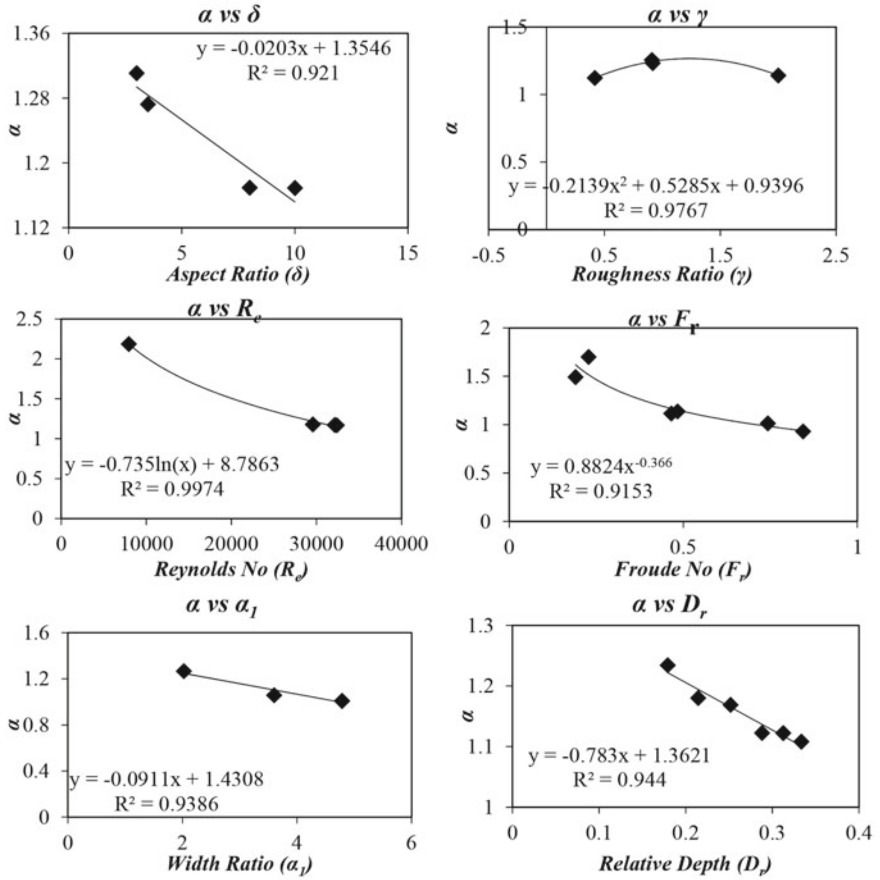

Fig. 26.5 Energy correction factor versus aspect ratio, roughness ratio, Reynolds number, Froude's number, width ratio, and relative depth plots with highest R^2 value

$$\text{MAPE} = \frac{100}{N}\sum_{i=1}^{N}\left|\frac{\alpha_{\text{observed}} - \alpha_{\text{predicted}}}{\alpha_{\text{observed}}}\right| \tag{26.8}$$

$$\text{MPE} = \frac{100}{N}\sum_{i=1}^{N}\left(\frac{\alpha_{\text{observed}} - \alpha_{\text{predicted}}}{\alpha_{\text{observed}}}\right) \tag{26.9}$$

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\alpha_{\text{observed}} - \alpha_{\text{predicted}}\right)^2} \tag{26.10}$$

where α_{observed} the observed value of energy correction is factor and $\alpha_{\text{predicted}}$ is the predicted value of energy correction factor using the improved model.

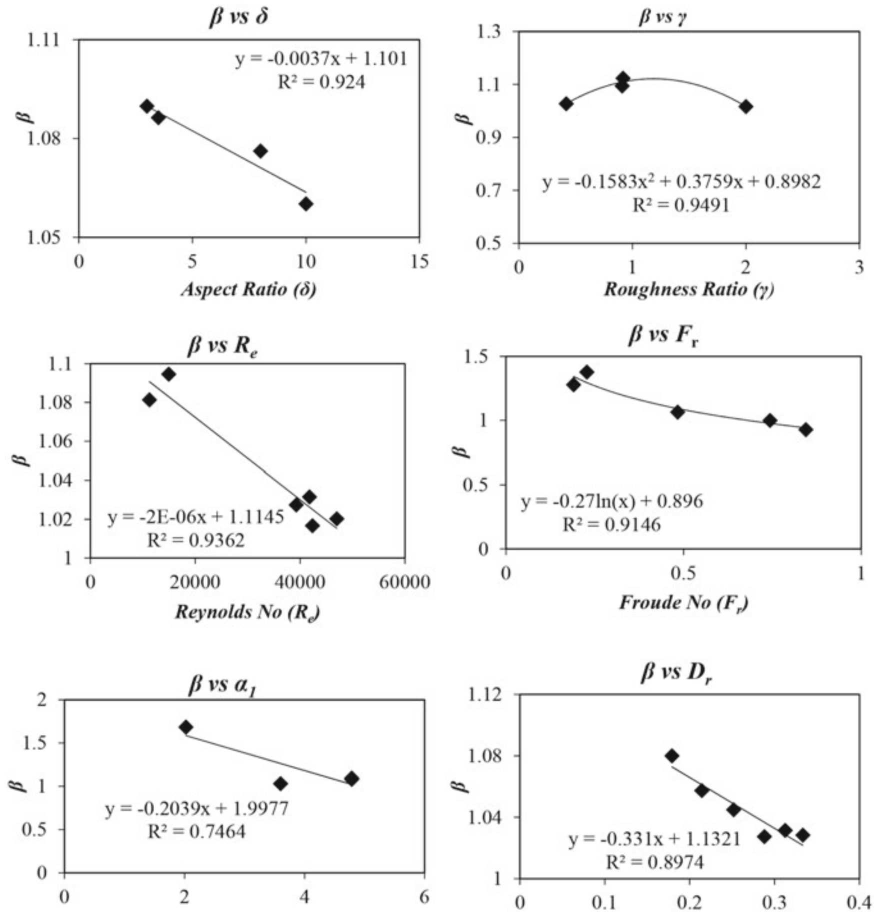

Fig. 26.6 Momentum correction factor versus aspect ratio, roughness ratio, Reynolds number, Froude's number, width ratio, and relative depth plots with highest R^2 value

Table 26.3 Relationships between dependent (α) and independent parameters (δ, γ, R_e, F_r, α_1, D_r) for energy correction factor

Parameter	Relation	Function	R^2
Aspect ratio (δ)	$\alpha = -0.0203\delta + 1.3546$	Linear	0.921
Roughness ratio(γ)	$\alpha = -0.2139\gamma^2 + 0.5285\gamma + 0.9396$	Polynomial	0.9767
Reynolds no (R_e)	$\alpha = -0.735\ln(R_e) + 8.7863$	Logarithmic	0.9974
Froude no (F_r)	$\alpha = 0.8824{F_r}^{-0.366}$	Power	0.9153
Width ratio (α_1)	$\alpha = -0.0911\alpha_1 + 1.4308$	Linear	0.9386
Relative depth (D_r)	$\alpha = -0.783D_r + 1.3621$	Power	0.944

Table 26.4 Relationships between dependent (β) and independent parameters (δ, γ, R_e, F_r, α_1, D_r) for energy correction factor

Parameter	Relation	Function	R^2
Aspect ratio (δ)	$\beta = -0.0037\delta + 1.101$	Linear	0.924
Roughness ratio(γ)	$\beta = -0.1538\gamma^2 + 0.3759\gamma + 0.8982$	Polynomial	0.9491
Reynolds no (R_e)	$\beta = -2E{-}06R_e + 1.1145$	Linear	0.9362
Froude no (F_r)	$\beta = -0.27\ln(F_r) + 0.896$	Logarithmic	0.9146
Width ratio (α_1)	$\beta = -0.2039\alpha_1 + 1.9977$	Linear	0.7464
Relative depth (D_r)	$\beta = -0.331D_r + 1.1321$	Linear	0.8974

Table 26.5 Resulted error from the present models

Correction factor	MPE (%)	MAPE (%)	RMSE
Energy correction coefficient (α)	−0.715	4.562	0.079
Momentum correction coefficient (β)	−0.358	2.897	0.023

The above equations (Eqs. 26.8–26.10) were used to predict the errors for momentum correction coefficient (β) by replacing α with β. The various types of errors are presented in Table 26.5.

From Table 26.5, it has been observed that the MPE is −0.715%, MAPE is 4.562% and RMSE is 0.079 for the model of energy correction coefficient (α) and for the model of momentum correction coefficient (β), the errors are −0.358%, 2.897% and 0.023 as MPE, MAPE and RMSE respectively. As all the error parameters are closer to zero, it has been concluded that both the models are being well predicted for asymmetrical compound channels.

26.7 Conclusions

1. Experimental analysis has been performed on the asymmetrical channel to analyse the correction factors (α and β).
2. It has been observed that the magnitudes of both the correction factors (α and β) decrease when the depth of flow increases.
3. The transfer of momentum between the main channel and flood plain becomes very intense in case of low flow depth conditions due more difference in flow velocities of main channel and flood plain. But while considering high flow depth cases, this difference becomes less.
4. The resulted energy and momentum coefficients were higher for low flow depth cases and nearer to one when the flow depth increased for the present experimental channel.

5. Improved models have been developed for both energy and momentum correction factors using multiple regression analysis. Error analysis was also performed for both the models.

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